Chapter 10: CLP Systems

Where we examine how CLP systems work and introduce an important concept for constraint solvers: incrementality

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CLP Systems

- Simple Backtracking Goal Evaluation
- Incremental Constraint Solving
- Efficient Saving and Restoring of the Constraint Store
- Implementing If-Then-Else, Once and Negation
- Optimization
- Other Incremental Constraint Solvers

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Backtracking Goal Evaln.

- Previously understood as depth-first left-right search through a derivation tree
- Specific algorithm: simple_solve_goal
  - parametric in *solv* and *simpl*
  - uses *defn(P,L)* which returns rules defining *L* in program *P* in the order they occur, renamed to not contain any previous variables
- simple_solve_goal(G)
  - **return** *simpl*(*vars*(G) , simple_backtrack(<G|*true*>})

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simple_backtrack

- simple_backtrack(<G|C>)
  - **if** *G* is empty **return** *C*
  - **let** *G* be of the form *L, G'*
  - **case** *L* is a primitive constraint
    - if *solv*($C \wedge L$) = *false* **return** *false*
    - **return** simple_backtrack(<*G'*|$C \wedge L$>)
  - **case** *L* is an atom *p(s1,...,sn)*
    - **foreach** *p(t1,...,tn)* :- *B* in *defn(P,L)*
      - *C1* = simple_backtrack(<*s1=t1,..,sn=tn,B,G'*|*C*>)
      - if *C1* != *false* **return** *C1*
    - **return** false

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Example execution sum(1,S)

```
(S1) sum(0,0).
(S2) sum(N,N+S) :- sum(N-1,S).
```

simple_backtrack(<*sum(1,S)* | *true*>)
  simple_backtrack(<*1=0,S=0* | *true*>)   rule S1
    returns *false*
  simple_backtrack(<*1=N',S=N'+S',sum(N'-1,S')*|*true*>) rule S2
    simple_backtrack(<*S=N'+S',sum(N'-1,S')*| *1=N'*>)
      simple_backtrack(<*sum(N'-1,S')*| $1=N' \wedge S=N'+S'$ >)
        simple_backtrack(<*N'-1=0,S'=0* | $1=N' \wedge S=N'+S'$ >) rule S1
          simple_backtrack(<*S'=0* | $1=N' \wedge S=N'+S' \wedge N'-1=0$ >)
            simple_backtrack(<*[]* | $1=N' \wedge S=N'+S' \wedge N'-1=0 \wedge S'=0$>)
            returns $1=N' \wedge S=N'+S' \wedge N'-1=0 \wedge S'=0$

simpl($\{S\}, 1=N' \wedge S=N'+S' \wedge N'-1=0 \wedge S'=0) = S = 1$

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Incremental Solving

- The simple backtracking evaluation is <u>inefficient</u>, consider calls to *solv*
  - *solv*$(1=0)$
  - *solv*$(1 = N')$
  - *solv*$(1= N' \wedge S = N + S')$
  - *solv*$(1= N' \wedge S = N + S' \wedge N'-1 = 0)$
  - *solv*$(1= N' \wedge S = N + S' \wedge N'-1 = 0 \wedge S' = 0)$
- Repeated work

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Incremental Constraint Solver

- An **incremental constraint solver** is a function *isolv* which takes a primitive constraint *c* and returns *true*, *false* or *unknown.* There is an implicit *constraint store* S
  - if $isolv(c) = true$ then $S \wedge c$ is satisfiable
  - if $isolv(c) = false$ then $S \wedge c$ is unsatisfiable
  - if $isolv(c)$!= *false* then store is updated to $S \wedge c$

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Incremental Gauss-Jordan

- inc_gj(*c*)
  - c := eliminate(c, S)
  - **if** *c* is of the form 0 = 0 **return** *true*
  - **if** *c* is of the form d = 0 (d != 0) **return** *false*
  - rewrite *c* in the form $x = e$
  - S := eliminate(S, x = e) $\wedge\, x = e$
  - **return** *true*
- eliminate($C, x1 = e1 \wedge \ldots \wedge xn = en$)
  - **foreach** *xi*
    - replace *xi* by *ei* throughout *C*
  - **return** *C*

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Incremental GJ Example

Solving $1 = N' \wedge S = N + S' \wedge N' - 1 = 0 \wedge S' = 0$

	S	c	eliminate(c,S)
$isolv(N'=1)$	*true*	$1 = N'$	$1 = N'$
$isolv(S=N'+S')$	$N' = 1$	$S = N' + S'$	$S = 1 + S'$
$isolv(N'-1 = 0)$	$N' = 1 \wedge S = 1 + S'$	$N' - 1 = 0$	$0 = 0$
$isolv(S' = 0)$	$N' = 1 \wedge S = 1 + S'$	$S' = 0$	$S' = 0$
	$N' = 1 \wedge S = 1 \wedge S' = 0$		

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Incremental goal solver

- CLP systems use a global constraint store *S* and incremental solvers
- inc_backtrack similar to simple_backtrack
  - uses incremental solver
  - store is not part of argument
  - functions: save_store, restore_store for saving and restoring the implicit store

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inc_backtrack

- inc_backtrack(G)
  - **if** *G* is empty **return** *true*
  - **let** *G* be of the form *L, G'*
  - **case** *L* is a primitive constraint
    - if i*solv*(*L*) = *false* **return** *false*
    - **return** inc_backtrack(*G'*)
  - **case** *L* is an atom *p(s1,...,sn)*
    - **foreach** *p(t1,...,tn)* :- *B* in *defn(P,L)*
      - save_store()
      - **if** inc_backtrack(*s1=t1,..,sn=tn,B,G'*) **then**
        - **return** *C1*
      - restore_store()
    - **return** *false*

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inc_solve_goal

- The incremental goal solving algorithm, making use of auxiliary functions to initialize and get the constraint store
- inc_solve_goal(*G*)
  - *W* := *vars(G)*
  - initialize_store()
  - **if** inc_backtrack(*G*) then
    - **return** *simpl*(*W*, get_store())
  - **return** *false*

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Example execution sum(1,S)

	constraint store stack
inc_backtrack(*sum(1,S)*)	*<empty>*
inc_backtrack(*1=0, S = 0*)	*true* |
return *false*	*<empty>*
inc_backtrack(*1=N', S = N' + S', sum(N'-1, S')*)	*true* |
inc_backtrack(*S = N' + S', sum(N'-1, S')*)	*true* |
inc_backtrack(*sum(N'-1, S')*)	*true* |
inc_backtrack(*N'-1 = 0, S' = 0*)	*true* | $N' = 1 \wedge S = 1 + S'$ |
inc_backtrack(*S' = 0*)	*true* | $N' = 1 \wedge S = 1 + S'$ |
inc_backtrack(*[]*)	*true* | $N' = 1 \wedge S = 1 + S'$ |

$simpl(\{S\}, N' = 1 \wedge S = 1 \wedge S' = 0) = S = 1$

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Efficient saving and restoring

- Incremental solver requires saving/restoring the constraint store
- Dont need to save the entire store
- Save enough information to recreate store
  - **Trailing**: save modified parts of the constraint store in a trail and recover on backtracking
  - **Semantic backtracking**: store operations necessary to recover store

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Trailing

- Associate a **timestamp** with each primitive constraint
- At a choicepoint
  - store the current timestamp
- Backtracking
  - remove all constraints with a later stamp
- Doesnt handle when an old primitive constraint is modified

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Trailing

- Whenever an old constraint (from before the last choicepoint) is modified
  - save the old value in the **trail**
- Note we dont have to trail the same constraint again if it is modified again before another choicepoint

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Trailing Gauss-Jordan

- Index each equation by arrival number
- Choicepoint saves:
  - index of last equation, *last*
  - trail of changes (initially empty)
- Whenever equation *i* is modified, if *i* <= *last* then each modified coefficient is added to trail <*i,x,a*> or <*i,constant,b*>

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Semantic Backtracking

- Save high-level operations of how to restore the constraint store (domain dependent)
- For Gauss-Jordan
  - a new constraint only eliminates a variable *x*
  - remember the old coefficients of *x* and
  - **undo** the elimination on backtracking

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Semantic Backtracking Ex.

Imagine store is

1: $X = \boxed{Y} + 2Z + 4$
2: $U = \boxed{3Y} + Z - 1$
3: $V = 3$

Removing constraint

Adding constraint $Y + 2V + X = 2$

Eliminate vars $Y = -Z - 4$

Add coefficient * (Y+Z+4) to eqns 1,2 and remove 4

Eliminate Y using equation and add. Remember coefs $[\langle 1, Y, 1\rangle, \langle 2, Y, 3\rangle]$

1: $X = Z$
2: $U = -2Z - 13$
3: $V = 3$
4: $Y = -Z - 4$

1: $X = Y + 2Z + 4$
2: $U = 3Y + Z - 1$
3: $V = 3$

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Extra Constructs

- So far "pure" programs (Chapter 4)
- Chapters 7 and 9 introduce
  - if-then-else
  - once
  - negation
  - optimization
- How are they implemented?

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If-Then-Else,Once+Negation

- All three are implemented using a single construct, the **cut**, written !
- Cut prunes derivations from a tree
  - when reached: commit to this clause and remove any choices set up within this clause
- Very powerful, and dangerous
- Preferable to use if-then-else, once or negation rather than the lower level cut

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Cut Example

Sum program for mode of usage: first arg fixed

```
sum(N,SS) :-
     (N = 0 ->
          SS = 0
     ;
          N >= 1, SS = N + S,
          sum(N-1,S)
     ).
```

Equivalent version with cut

```
sum(N,SS) :- N = 0, !, SS = 0.
sum(N,SS) :- N >= 1, SS = N + S,
                sum(N-1, S).
```

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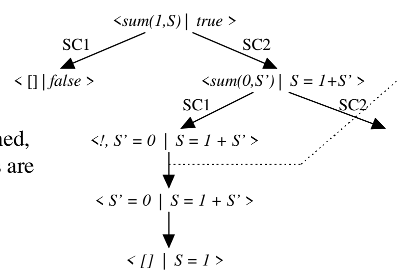
Cut Derivation Tree



When ! reached, other choices are pruned away

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Cut

- Cut commits to all choices made since when the atom which was rewritten that introduced the cut
- Assume rewriting atom *A'* using rule
  - *A* :- *L1, ..., Li, !, Li+1, ..., Ln*
- When ! reached all choices for rewriting *A'* and all choices in evaluation *L1*, ..., *Li* are removed

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Implementing Cut

- Need save_store to return an index of the last store
- remove_choicepoints(*i*) removes all choicepoints with indexes >= *i*
- simply modify inc_backtrack for case introducing a cut

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Modifying inc_backtrack

- **case** *L* is an atom *p(s1,...,sn)*
  - **foreach** *p(t1,...,tn)* :- *L1,...,Ln* in *defn(P,L)*
    - *i* := save_store()
    - **if** some *Lj* = ! **then**
      - **if** inc_backtrack(*s1=t1,..,sn=tn,L1,...,Lj-1)* **then**
        - remove_choicepoints(i)
        - **return** inc_backtrack(*Lj+1,...,Ln,G'*)
    - **elseif** inc_backtrack(*s1=t1,..,sn=tn,,G'*) then
      - **return** *true*
    - restore_store()
  - **return** *false*

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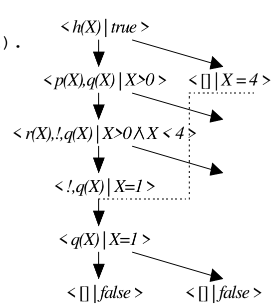
Cut Example 2

```
h(X):- X > 0,p(X),q(X).
h(4).
p(X) :-X < 4, r(X),!.
p(3).
r(1).
r(2).
q(2),
q(3).
```



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Cut Example 2

		constraint storestack
inc_backtrack($h(X)$)		*<empty>*
inc_backtrack($X > 0, p(X), q(X)$)		$true \mid$
inc_backtrack($p(X), q(X)$)	**index 2**	$true \mid X > 0 \mid$
inc_backtrack($X < 4, r(X)$) (before cut)		$true \mid X > 0 \mid$
inc_backtrack($r(X)$)		$true \mid X > 0 \mid X > 0 \wedge X < 4 \mid$
return *true*	**remove upto 2**	$true \mid$
inc_backtrack($q(X)$) (after cut)		$true \mid X = 1 \mid$
return *false*	**restore 2**	$true \mid$
return *false*	**restore 1**	*<empty>*
inc_backtrack($X = 4$)		$true \mid$
return *true*	Answer: $X = 4$	

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If-Then-Else,Once+Negation

All are implemented using the meta-programming facilities and the cut.

```
once(G) :- call(G), !.

not(G) :- call(G)!, !, fail.
not(G).

G1 -> G2 ; G3 :- call(G1), !, call(G2).
G1 -> G2 ; G3 :- call(G3).
```

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Optimization

- Implementing `minimize(G,E)`
  - minimize_store(E): returns the minimal value of E wrt to current constraint store
  - search the derivation tree of G and collect minimum value m of E, then execute $E = m, G$
- Multiple approaches to search
  - retry search (restart after finding soln)
  - backtrack search (continue after finding)

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Retry Optimization

- **case** *L* is minimization literal *minimize(G,E)*
  - i := save_store()
  - m := + ∞
  - **while** inc_backtrack($E < m, G$) **do**
    - m := minimize_store(E)
    - remove_choicepoints($i+1$)
    - restore_store()
    - i := save_store()
  - restore_store()
  - **return** inc_backtrack($E = m, G, G'$)

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Retry Example

Evaluating `minimize(butterfly(S,P), -P)`

inc_backtrack($-P < +\infty$, *butterfly(S,P)*)
  answer: $-P < +\infty \wedge P = -100 \wedge 0 <= S <= 1$
  m := 100
inc_backtrack($-P < 100$, *butterfly(S,P)*)
  answer: $-P < 100 \wedge P = 100S - 200 \wedge 1 <= S <= 3$
  m := -100
inc_backtrack($-P < -100$, *butterfly(S,P)*)
  returns *false*
inc_backtrack($-P = -100$, *butterfly(S,P)*)
  answers: $P = 100 \wedge S = 3$ (twice)

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Backtracking Optimization

- `minimize(G,E)`
- Search the derivation tree for *G*
- At each success update the minimal value *m* of *E* found (handled by a *catch* literal)
- Then execute *E=m,G*

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Backtracking Optimization

- **case** L is minimization literal *minimize(G,E)*
  - i := save_store()
  - $m := +\infty$
  - inc_backtrack(G, *catch(m,E)*)
  - restore_store(i)
  - **return** inc_backtrack(*E=m,G,G'*)
- **case** L is a catch subgoal *catch(m,E)*
  - **if** *isolv*($E < m$) != *false* **then**
    - m := minimize_store(E)
  - **return** *false*

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Backtracking Example

inc_backtrack(*butterfly(S,P),catch(+   ,-P)*)
  inc_backtrack(*catch(+   ,-P)*)
    store: $P = -100 \wedge 0 <= S <= 1$ **sets** $m :=$ 100
  inc_backtrack(*catch(100,-P)*)
    store: $P = 100S - 200 \wedge 1 <= S <= 3$ **sets** $m :=$ -100
  inc_backtrack(*catch(-100,-P)*)
    store: $P = -100S + 400 \wedge 3 <= S <= 5$
    isolv(-P < -100) **fails** no update
  inc_backtrack(*catch(100,-P)*)
    store: $P = -100\ S >= 5$ **fails** no update
inc_backtrack(*-P = -100, butterfly(S,P)*)
  answers: $P = 100 \wedge S = 3$ (twice)

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Other Incremental Solvers

- Incremental Tree Solving
  - Use the store to eliminate variables and solve remainder as before, then use it to eliminate
  - inc_tree_solve(c)
    - c := eliminate(c, S)
    - R := unify(c)
    - **if** R = *false* **then return** *false*
    - S := eliminate(S,R) $\wedge$ R
    - **return** *true*

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Incremental Tree Solving Ex.

Constraints collected by goal `append([a],[b,c],L)`

$[a] = [F|R]$, $[b,c] = Y$, $L = [F|Z]$, $R = []$, $Y = Z$

c	S	elim(c)	unify(c)
$[a]=[F\|R]$	*true*	$[a]=[F\|R]$	$F=a \wedge R=[]$
$[b,c]=Y$	$F=a \wedge R=[]$	$[b,c]=Y$	$Y=[b,c]$
$L=[F\|Z]$	$F=a \wedge R=[] \wedge Y=[b,c]$	$L=[a\|Z]$	$L=[a\|Z]$
$R=[]$	$F=a \wedge R=[] \wedge Y=[b,c] \wedge L=[a\|Z]$	$[]=[]$	*true*
$Y=Z$	$F=a \wedge R=[] \wedge Y=[b,c] \wedge L=[a\|Z]$	$[b,c]=Z$	$Z=[b,c]$
	$F=a \wedge R=[] \wedge Y=[b,c] \wedge L=[a,b,c] \wedge Z=[b,c]$		

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Data Structures for Trees

- Tree constraints are stored/manipulated as dynamic data structures
  - **variable**: unique memory cell (pointer)
    - unconstrained: self-pointer
    - equated to term: pointer at term rep
  - **term** *f(t1,...,tn)*: *n+1* memory cells
    - first: constructor info *f/n*
    - rest: pointers to *t1,...,tn*
  - optimization: store subtrees with no children directly

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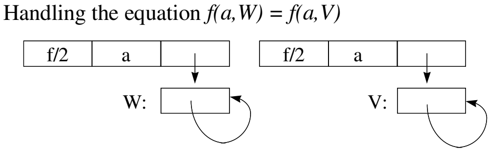
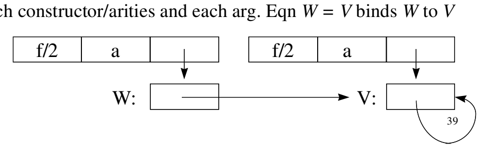
Data Structures for Trees

Handling the equation *f(a,W) = f(a,V)*

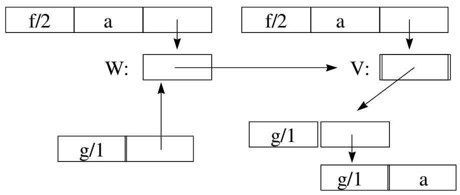


Match constructor/arities and each arg. Eqn $W = V$ binds W to V



Data Structures for Trees

Incrementally adding $g(W) = g(g(a))$



Represents solved form: $V = g(a) \wedge W = g(a)$

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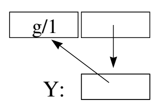
Occurs Check Revisited

- Most implementations ignore the occurs check!
- Problems: e.g. $Y = g(Y)$
- Builds cyclic structures
- Infinite computation
- e.g. $Y = g(Y), Z = g(Z), Y = Z$



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Incremental Bounds Cons.

- Propagation is essentially incremental
- incremental bounds consistency:
  - Add new prim. constraint to store and queue
  - Pick prim. constraint from queue
  - Enforce its bounds consistency
  - Add prim. constraint with modified variables to queue
  - Repeat until queue is empty, or empty domain

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Incremental Bounds Ex.

Smugglers knapsack, no whiskey

$$\underbrace{\boxed{4W + 3P + 2C \le 9}}_{\text{capacity}} \wedge \underbrace{\boxed{15W + 10P + 7C \ge 30}}_{\text{profit}}$$

$D(W) = [0..0], D(P) = [1..3], D(C) = [0..3]$

Add first constraint

Add second constraint

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CLP Systems Summary

- Incremental constraint solving
  - essential for efficiency
- Global constraint store
  - require efficient save and restore
- The Cut!
  - implements if-then-else, once + negation
- Minimization
  - many possible implementations

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