Chapter 6: Using Data Structures

Where we find how to use tree constraints to store and manipulate data

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Using Data Structures

- Records
- Lists
- Association Lists
- Binary Trees
- Hierarchical Modelling
- Tree Layout

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Records

- simplest type of data structure is a record
- **record** packages together a fixed number of items of information, often of different type
- e.g. *date(3, feb, 1997)*
- e.g. complex numbers $X + Yi$ can be stored in a record *c(X, Y)*

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Complex Numbers

Complex number *X + Yi* is represented as *c(X,Y)*

Predicates for addition and multiplication

```
c_add(c(R1,I1), c(R2,I2), c(R3,I3)) :-
     R3 = R1 + R2, I3 = I1 + I2.
c_mult(c(R1,I1), c(R2,I2), c(R3,I3)) :-
     R3 = R1*R2 - I1*I2, I3 = R1*I2 + R2*I1.
```

Note they can be used for subtraction/division

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Example Adding *1+3i* to *2+Yi*

$$\langle C1 = c(1,3), C2 = c(2,Y), c\_add(C1,C2,C3) \mid true \rangle$$
$$\Downarrow$$
$$\langle c\_add(C1,C2,C3) \mid C1 = c(1,3) \wedge C2 = c(2,Y) \rangle$$
$$\Downarrow$$
$$\langle C1 = c(R1,I1), C2 = c(R2,I2), C3 = c(R3,I3), R3 = R1 + R2, I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \rangle$$
$$\Downarrow$$
$$\langle C2 = c(R2,I2), C3 = c(R3,I3), R3 = R1 + R2, I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \rangle$$
$$\Downarrow$$
$$\langle C3 = c(R3,I3), R3 = R1 + R2, I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \wedge \underline{R2 = 2 \wedge I2 = Y} \rangle$$

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Example Adding *1+3i* to *2+Yi*

$$\langle C1 = c(1,3), C2 = c(2,Y), c\_add(C1,C2,C3) \mid true \rangle$$
$$\Downarrow^{*}$$
$$\langle C3 = c(R3,I3), R3 = R1 + R2, I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \wedge R2 = 2 \wedge I2 = Y \rangle$$
$$\Downarrow$$
$$\langle R3 = R1 + R2, I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \wedge R2 = 2 \wedge I2 = Y \wedge C3 = c(R3,I3) \rangle$$
$$\Downarrow$$
$$\langle I3 = I1 + I2 \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \wedge R2 = 2 \wedge I2 = Y \wedge C3 = c(R3,I3) \wedge R3 = 3 \rangle$$
$$\Downarrow$$
$$\langle [] \mid C1 = c(1,3) \wedge C2 = c(2,Y) \wedge R1 = 1 \wedge I1 = 3 \wedge R2 = 2 \wedge I2 = Y \wedge C3 = c(R3,I3) \wedge R3 = 3 \wedge I3 = 3 + Y \rangle$$

Simplifying wrt *C3* and *Y* gives

$$C3 = c(3, 3 + Y)$$

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Records

- Term equation can
  - build a record *C3 = c(R3, I3)*
  - access a field *C2 = c(R2, I2)*
- underscore _ is used to denote an **anonymous variable**, each occurence is different. Useful for record access
  - *D = date(_, M, _)* in effect sets *M* to equal the month field of *D*

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Lists

- Lists store a variable number of objects usually of the same type.
- empty list [] (list)
- list constructor . (item x list -> list)
- special notation:

$$
\begin{array}{cc}
[X|Y] & .(X,Y) \\
[X1, X2, \ldots, Xm|Y] & .(X1,.(X2,.(\cdots.(Xm,Y)))) \\
[X1, X2, \ldots, Xm] & .(X1,.(X2,.(\cdots.(Xm,[]))))
\end{array}
$$

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List Programming

- Key: reason about two cases for *L*
  - the list is empty *L = []*
  - the list is non-empty *L = [F|R]*
- Example concatenating *L1* and *L2* giving *L3*
  - *L1* is empty, *L3* is just *L2*
  - *L1* is *[F|R]*, if *Z* is *R* concatenated with *L2* then *L3* is just *[F|Z]*

```
append([], L2, L2).
append([F|R], L2, [F|Z]) :- append(R,L2,Z).
```

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Concatenation Examples

```
append([], L2, L2).
append([F|R], L2, [F|Z]) :- append(R,L2,Z).
```

- concatenating lists `append([1,2],[3,4],L)`
- has answer $L = [1,2,3,4]$
- breaking up lists `append(X,Y,[1,2])`
- ans $X=[] \wedge Y=[1,2], X=[1] \wedge Y=[2], X=[1,2] \wedge Y=[]$
- BUT is a list equal to itself plus [1]
- `append(L,[1],L)` runs forever!

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Alldifferent Example

We can program alldifferent using disequations

```
alldifferent_neq([]).
alldifferent_neq([Y|Ys]) :-
  not_member(Y,Ys), alldifferent_neq(Ys).
not_member(_, []).
not_member(X, [Y|Ys]) :-
     X != Y, not_member(X, Ys).
```

The goal `alldifferent_neq([A,B,C])` has one solution $A \neq B \wedge A \neq C \wedge B \neq C$

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Arrays

- Arrays can be represented as lists of lists
- e.g. a 6 x 7 finite element description of a metal plate 100C at top edge 0C other edges

```
[[0, 100, 100, 100, 100, 100, 0],
 [0,   _,   _,   _,   _,   _, 0],
 [0,   _,   _,   _,   _,   _, 0],
 [0,   _,   _,   _,   _,   _, 0],
 [0,   _,   _,   _,   _,   _, 0],
 [0,   0,   0,   0,   0,   0, 0]]
```

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Arrays Example

- In a heated metal plate each point has the average temperature of its orthogonal neighbours

```
rows([_,_]).
rows([R1,R2,R3|Rs]) :-
     cols(R1,R2,R3), rows([R2,R3|Rs]).
cols([_,_], [_,_], [_,_]).
cols([TL,T,TR|Ts],[L,M,R|Ms],[BL,B,BR|Bs]):-
     M = (T + L + R + B)/4,
     cols([T,TR|Ts],[M,R|Ms],[B,BR|Bs]).
```

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Arrays Example

- The goal `rows(`*plate*`)` constrains *plate* to

```
[[0,  100,  100,  100,  100,  100, 0],
 [0, 46.6, 62.5, 66.4, 62.5, 46.6, 0],
 [0, 24.0, 36.9, 40.8, 36.9, 24.0, 0],
 [0, 12.4, 20.3, 22.9, 20.3, 12.4, 0],
 [0,  5.3,  9.0, 10.2,  9.0,  5.3, 0],
 [0,    0,    0,    0,    0,    0, 0]]
```

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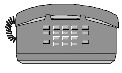
Association Lists

- A list of pairs is an **association list**
- we can access the pair using only one half of the information
- e.g. telephone book
  [*p(peter,5551616),*
  p(kim, 5559282),
  p(nicole, 5559282)]
- call this *phonelist*



peter	5551616
kim	5559282
nicole	5559282

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List Membership

```
member(X, [X|_]).
member(X, [_|R]) :- member(X, R).
```

X is a member of a list if it is the first element or it is a member of the remainder *R*

We can use it to look up Kims phone number

`member(p(kim,N),` *phonelist*`)`

Unique answer: *N = 5559282*

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List Membership Example

$$
\begin{array}{c|c}
 & \langle member(p(k,N),[p(p,161),p(k,282),p(n,282)])|true\rangle \\
\Downarrow E1 & \Downarrow E2 \\
\langle [] | p(k,N) = p(p,616)\rangle & \langle member(p(k,N),[p(k,282),p(n,282)])|true\rangle \\
\hline
\Downarrow E1 & \Downarrow E2 \\
\langle [] | p(k,N) = p(k,282)\rangle & \langle member(p(k,N),[p(n,282)])|true\rangle \\
\hline
\Downarrow E1 & \Downarrow E2 \\
\langle [] | p(k,N) = p(n,282)\rangle & \langle member(p(k,N),[])|true\rangle \\
\hline
\Downarrow E1 & \Downarrow E2 \\
\langle [] | false\rangle & \langle [] | false\rangle
\end{array}
$$

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Abstract Datatype: Dictionary

- *lookup* information associated with a key
- *newdic* build an empty association list
- *add key* and associated information
- *delete key* and information

```
lookup(D,Key,Info):-member(p(Key,Info),D).

newdic([]).

addkey(D0,K,I,D) :- D = [p(K,I)|D0].

delkey([],_,[]).
delkey([p(K,_)|D],K,D).
delkey([p(K0,I)|D0],K,[p(K0,I)|D]) :-
     K != K0, delkey(D0,K,D).
```

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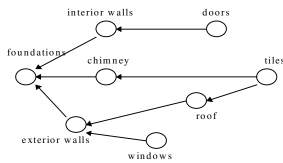
Modelling a Graph

- A directed graph can be thought of as an association of each node to its list of adjacent nodes.

[p(fn,[]), p(iw,[fn]),
p(ch,[fn]), p(ew,[fn]),
p(rf,[ew]), p(wd,[ew]),
p(tl,[ch,rf]), p(dr,[iw])]



call this *house*

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Finding Predecessors

The predecessors of a node are its immediate predecessors plus each of their predecessors

```
predecessors(N,D,P) :-
     lookup(D,N,NP),
     list_predecessors(NP,D,LP),
     list_append([NP|LP],P).
list_predecessors([],_,[]).
list_predecessors([N|Ns],D,[NP|NPs]) :-
     predecessors(N,D,NP),
     list_predecessors(Ns,D,NPs).
```

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Finding Predecessors

```
list_append([],[]).
list_append([L|Ls],All) :-
     list_append(Ls,A),
     append(L,A,All).
```

Appends a list of lists into one list.

We can determine the predecessors of tiles (*tl*) using:

`predecessors(tl,` *house*`, Pre)`

The answer is *Pre = [ch, rf, fn, ew, fn]*

Note repeated discovery *of fn*

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Accumulation

- Programs building an answer sometimes can use the list answer calculated so far to improve the computation
- Rather than one argument, the answer, use two arguments, the answer so far, and the final answer.
- This is an **accumulator pair**

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Finding Predecessors

- A better approach *accumulate* the predcsrs.

```
predecessors(N,D,P0,P) :-
     lookup(D,N,NP),
     cumul_predecessors (NP,D,P0,P).
cumul_predecessors([],_,P,P).
cumul_predecessors([N|Ns],D,P0,P) :-
     member(N,P0),
     cumul_predecessors(Ns,D,P0,P).
cumul_predecessors([N|Ns],D,P0,P) :-
     not_member(N,P0),
     predecessors(N,D,[N|P0],P1),
     cumul_predecessors(Ns,D,P1,P).
```

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Binary Trees

- empty tree: *null*
- non-empty: *node(t1, i, t2)* where *t1* and *t2* are trees and *i* is the item in the node
- programs follow a pattern (as for lists)
  - a rule for empty trees
  - a recursive rule (or more) for non-empty trees

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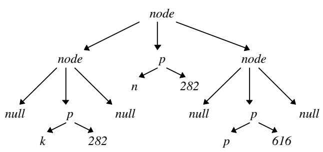
Binary Trees

node(node(null,p(k,282),null),p(n,282),node(null,p(p,616),null))



A binary tree storing the same info as *phonelist* denote it by *ptree*

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```
traverse(null,[]).
traverse(node(T1,I,T2),L) :-
     traverse(T1,L1),
     traverse(T2,L2),
     append(L1,[I|L2],L).
```

Program to traverse a binary tree collecting items

`traverse(`*ptree*`,L)`

has unique answer *L* =
[p(k,282),p(n,282),p(p,616)]

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Binary Search Tree

- **binary search tree** (BST): A binary tree with an order on the items such that for each *node(t1,i,t2)*, each item in *t1* is less than *i*, and each item in *t2* is greater then *i*
- previous example is a bst with right order
- another implementation of a dictionary!

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Binary Search Tree

Finding an element in a binary search tree

```
find(node(_,I,_),E) :- E = I.
find(node(L,I,_),E):-less_than(E,I),find(L,E).
find(node(_,I,R),E):-less_than(I,E),find(R,E).
```

Consider the goal `find(`*ptree*`, p(k,N))` with definition of `less_than` given below

```
less_than(p(k,_),p(n,_)).
less_than(p(k,_),p(p,_)).
less_than(p(n,_),p(p,_)).
```

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Binary Search Tree

$$\langle find(ptree, p(k,N)) | true \rangle$$
$$\Downarrow$$
$$\langle less\_than(p(k,N), p(n,282)), find(node(null, p(k,282), null), p(k,N)) | true \rangle$$
$$\Downarrow$$
$$\langle find(p(k,N), node(null, p(k,282), null)) | true \rangle$$
$$\Downarrow$$
$$\langle [] | N = 282 \rangle$$

The binary search tree implements a disctionary with logarithmic average time to lookup and add and delete

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Hierarchical Modelling

- Many problems are hierarchical in nature
- complex objects are made up of collections of simpler objects
- modelling can reflect the hierarchy of the problem

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Hierarchical Modelling Ex.

- steady-state RLC electrical circuits
  - sinusoidal voltages and currents are modelled by complex numbers:
  - individual circuit elements are modelled in terms of voltages and current:
  - circuits are modelled by combining circuit components

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Hierarchical Modelling Ex

- Represent voltages and currents by complex numbers: *V = c(X, Y)*
- Represent circuit elements by tree with component value: *E = resistor(100), E = capacitor(0.1), E = inductor(2)*
- Represent circuits as combinations or single elements: *C = parallel(E1, E2), C*

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Hierarchical Modelling Ex.

```
resistor(R,V,I,_) :- c_mult(I,c(R,0),V).
inductor(L,V,I,W) :- c_mult(c(0,W*L),I,V).
capacitor(C,V,I,W) :- c_mult(c(0,W*C),V,I).
circ(resistor(R),V,I,W):-resistor(R,V,I,W).
circ(inductor(L),V,I,W):-inductor(L,V,I,W).
circ(capacitor(C),V,I,W):-capacitor(C,V,I,W).
circ(parallel(C1,C2),V,I,W) :-c_add(I1,I2,I),
     circ(C1,V,I1,W),circ(C2,V,I2,W).
circ(series(C1,C2),V,I,W) :- c_add(V1,V2,V),
     circ(C1,V1,I,W),circ(C2,V2,I,W).
```

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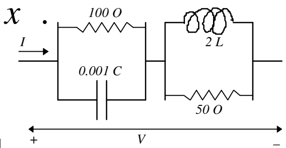
Hierarchical Modelling Ex.



The goal

circ(series(parallel(resistor(100),capacitor(0.0001)), parallel(parallel(inductor(2)),resistor(50)))),V,I,60).

gives answer

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Tree Layout Example

- Drawing a good tree layout is difficult by hand. One approach is using constraints
  - Nodes at the same level are aligned horizontal
  - Different levels are spaced 10 apart
  - Minimum gap 10 between adjacent nodes on the same level
  - Parent node is above and midway between children
  - Width of the tree is minimized

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Tree Layout Example

- We can write a CLP program that given a tree finds a layout that satisfies these constraints
  - a association list to map a node to coords
  - predicates for building the constraints
  - predicate to calculate width
  - a minimization goal

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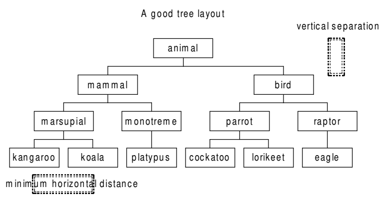
Tree Layout Example

node(node(node(node(null,kangaroo,null),marsupial,node(null,koala,null)),mammal,node(null,monotreme,node(null,platypus,null))),animal,node(node(node(null,cockatoo,null),parrot(node(null,lorikeet,null)),bird,node(null,raptor,node(null,eagle,null))))



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Data Structures Summary

- Tree constraints provide data structures
  - accessing and building in the same manner
- Records, lists and trees are straightforward
- Programs reflect the form of the data struct.
- Association lists are useful data structure for attaching information to objects
- Hierarchical modelling

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